Constrained and Simplified Gassmann's Equations

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Summary

For isotropic, homogeneous porous materials, Gassmann's (1951) equations are physically constrained by the Voigt and Reuss bounds and critical porosity. . These bounds provide stricter constraints on input parameters and output results of fluid saturation effects on bulk modulus. We recast the Gassmann's relations in terms of a porosity-dependent normalized modulus Kn, and pore fluid sensitivity in terms of a rock gain function G. These simplified Gassmann's relations suggest that correct input of grain bulk modulus and fluid modulus is key for accurate estimation of fluid saturation effects. We have developed new empirical relation (Han & Batzle 2000) to calculate fluid properties. Measured data on sandstone suggest that the gain function for reservoir sandstones (porosity of 20-30%) is around 2 and increases with increasing clay content. We also empirically estimate grain bulk modulus for clean and shaly sandstone based on measured velocities.

Introduction

Gassmann's (1951) equations are the most widely used relations to calculate seismic velocity changes due to different fluid saturations in reservoirs. These equations are predominant in the analysis of direct hydrocarbon indicators (DHI) such as amplitude 'bright spots', amplitude versus offset (AVO) as well as for time-lapse reservoir monitoring. Despite the popularity of Gassmann's equations and their incorporation within most software packages for seismic reservoir interpretation, important aspects of these equations have not been thoroughly examined. Many efforts have been made to understand the operation and application of Gassmann's equations (Mavko and Mukerji, 1995, Mavko, et al., 1998, Sengupta, and Movko., 1999, Nolen-Hoeksema, 2000). Most these works have attempted to isolate individual parameter effects.

Because the full implications of parameter interactions and interrelationships are not well understood, in general practice, there are few constraints placed on input parameters and there is little quality control of results. This is a particular problem for automated analysis in which results are usually taken at face value. In this paper, we will discuss typical bounds to provide stricter constraints on the input parameters and output results, formulate simplified Gassmann's equation, examine the nature of pore fluid sensitivity (gain function) and provide application tips.

Modulus and Gassmann's Equation

Typical P- and S-wave velocities measured on dry and water saturated sands are shown in Figure 1.

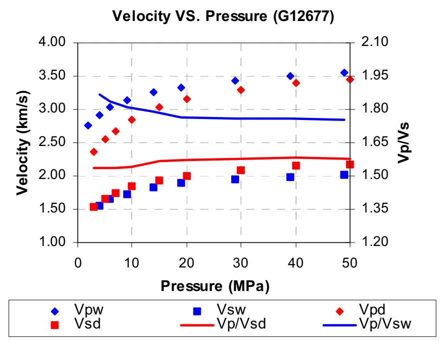

Fig. 1. Measured Vp and Vs versus differential pressure on dry and water saturated sands.

Usually, upon water saturation from the dry state, P-wave increases slightly and S-wave decrease slightly. The velocity response to water saturation is rather complex and physically controlled by the influence of the pore fluid on the rock moduli (equation 1).

$$V_p = \sqrt{\frac{K + 4/3\mu}{\rho}} \quad ; \quad V_s = \sqrt{\frac{\mu}{\rho}} \qquad (1)$$

However, if we plot bulk and shear modulus as shown in Figure 2, we can clearly separate fluid saturation effects on bulk and shear moduli.

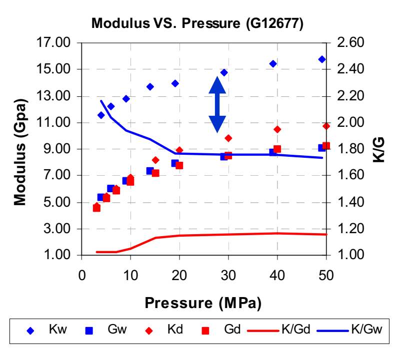

Fig. 2. Bulk and shear modulus based on measured velocities on dry and water saturated sands.

Typically we see a very large increase of bulk modulus is due to water saturation and negligible effect on shear modulus. This reflects the fact that bulk deformation produces a pore volume change (and thus a fluid pressure change), but shear deformation does not. Therefore, pore fluids effectively stiffen the rock frame and bulk modulus, but does not influence shear rigidity (assuming the fluid has negligible viscosity). Although bulk modulus is not directly measured in seismic data, we should use this modulus as a more direct fluid indicator.

Gassmann's equations describe the fluid saturation effect at 'zero' frequency. They provide a low bound of fluid saturation effects on bulk modulus under undrained conditions. We have shown that for porous sands, Gassmann's equation is normally valid (Batzle and Han, 2001) and can be expressed as:

$$K_{sat} = K_{dry} + \Delta K$$

$$\Delta K = \frac{(K_s - K_{dry})^2}{K_s \times (1 - \phi - \frac{K_{dry}}{K_s} + \phi \times \frac{K_s}{K_f})} \quad (2)$$

Rigidity is not sensitive to different pore fluids

$$\mu_{sat} = \mu_{dry} \quad (3)$$

Here K_s, K_f, K_{dry}, K_{sat}, are the bulk moduli of the mineral, fluid, dry rock frame, and saturated rock respectively; ϕ is porosity; and μ_{sat} and μ_{dry} are the saturated and dry rock shear moduli.

Constraints on Gassmann's Equation

The basic assumptions for Gassmann's equation do not provide strong constraints rock parameters. In the equations (1), there are five listed parameters and typically the only applied constraint in fluid substitution investigations is that the parameters are physically meaningful (>0). When applying Gassmann's equation, input parameters in general are handled as completely independent. Values for K_s and K_f are estimated or assumed. K_{sat} or K_{dry} are calculated from Vp and Vs, and density, which come from log data or are somehow estimated from porosity ϕ. Incompatible or mismatched data often generate wrong or even unphysical results, such as a negative modulus. In reality, only K_s and K_f, are completely independent. K_{sat}, K_{dry} and porosity ϕ are actually closely correlated. Bounds on K_{dry} as a function of porosity, for example, constrain the bounds of K_{sat}.

If we assume the porous media is a Voigt material, which is a high bound for $K_{dry,}$

$$K_{dry} = K_s \times (1 - \phi) \quad (4)$$

Putting Equation (4) into Equation (2) gives

$$\Delta K = \phi \times K_f \quad (5)$$

Since the Voigt bound is the stiffest upper limit, the fluid saturation effect on bulk modulus will be a minimum as shown in Figure 3. This is the first constrain derived from the Gassmann's equation: the minimum of bulk modulus increment due to fluid saturation is proportional to porosity of rock and modulus of pore fluid.

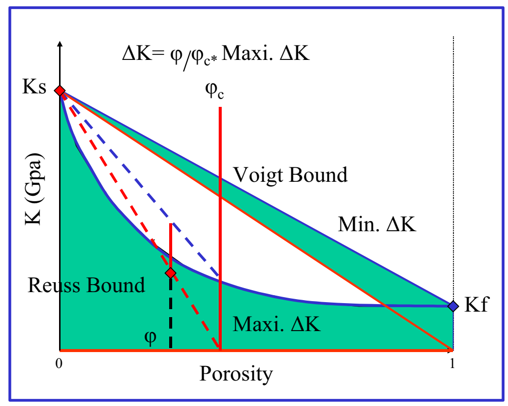

Fig. 3. The Voigt and Reuss bounds for dry rock, fluid saturated rock and fluid saturation effect predicted by the Gassmann's equation.

The Reuss bound is the lowest modulus bound for porous media:

$$\frac{1}{K_R} = \frac{(1-\phi)}{K_s} + \frac{\phi}{K_f} \quad (6)$$

For any porosity, the Reuss bound on K_{dry} is equal to zero. In this case,

$$K_{sat} = \Delta K = \frac{K_s}{1 - \phi + \phi \times {K_s}/{K_f}} = K_R \quad (7)$$

Again, Gassmann's equation is consistent with the dry and fluid saturated Reuss bounds. This Reuss bounded ΔK (K_R) is the maximum Gassmann's fluid saturation effect as shown in Figure 3.

Based on the critical porosity concept (Nur, 1995), we can modify our Voigt model (Figure 3) and provide much tighter constrains for dry and fluid saturated bulk modulus.

This Voigt triangle provides a linear formulation and a graphic procedure for Gassmann's calculation. The increment of bulk modulus is here proportional to normalized porosity and the maximum fluid saturation effect on bulk modulus at the critical porosity (Figure 3). This is consistent to earlier work by Mavko et al., (1995).

Simplified Gassmann's Equation

Gassmann's formulation is already very simple as shown in equation 2 and 3. This is a prime reason for its wide application in geophysical techniques for reservoir exploration and exploitation. However, derivation of the rock and fluid input parameters often remains ambiguous. In this section, we regroup Gassmann's equation with combined rock parameters. Under certain conditions we can further simplify this equation. This then separates the influence of fluid saturation into a lithology or textural component and a fluid modulus component.

The primary measure of a rock's compressibility is its normalized modulus Kn: the ratio of dry bulk modulus to that of the mineral.

$$K_n = K_{dry} / K_s \qquad (8)$$

The normalized modulus can be very complicated depending on rock texture (porosity, clay content, pore geometry, grain size, grain contact, cementation, mineral composition, and so on) and reservoir conditions (pressure and temperature). The K_n can be ascertained empirically, or estimated theoretically. To a first order, the K_n (x, y, z,…) can be simplified as a function of porosity.

$$K_n(x, y, z, ...) \cong K_n(\phi) \qquad (9)$$

From equation (3), bulk modulus increment is then equal to

$$\Delta K = \frac{K_s (1 - K_n(\phi))^2}{1 - \phi - K_n(\phi) + \phi \times \dfrac{K_s}{K_f}} \qquad (10)$$

Furthermore, since usually $K_s >> K_f$, for reservoir rocks (ϕ>0.1)

$$0 \le 1 - \phi - K_n(\phi) << \phi \times K_s / K_f \qquad (11)$$

Therefore,

$$\Delta K \le G(\phi) \times K_f \qquad (12)$$

where G(ϕ) is dry frame properties called the saturation Gain function and defined as

$$\frac{\Delta K}{K_f} \le G(\phi) = \frac{(1 - K_n(\phi))^2}{\phi} \qquad (13)$$

Equation 12 is a simplified form of Gassmann's equation with clear physical meaning: fluid effects on the rock bulk modulus are simply proportional to Gain function G(ϕ) of dry rock and decomposed fluid modulus K_f. The G(ϕ) in turn depends on the normalized modulus (Equation 8) and porosity. Equation 13 shows that the normalized modulus must compatible with porosity. Figure 4 shows upper and low bounds of gain function. Calculated gain function from velocity data on sandstones (Han, et al, 1986) is consistent with the bound (Figure 5).

Constrain of Gain Function

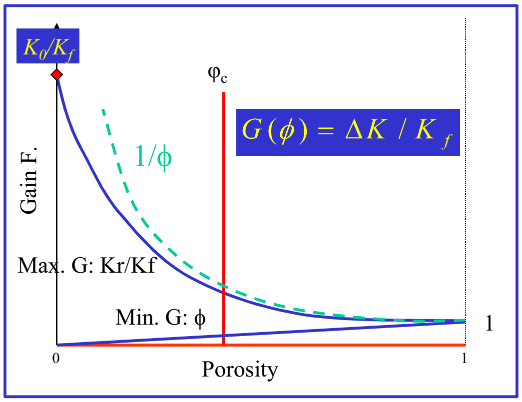

Fig. 4. Constrain of gain function by the Voigt and Reuss bounds

Gain Function

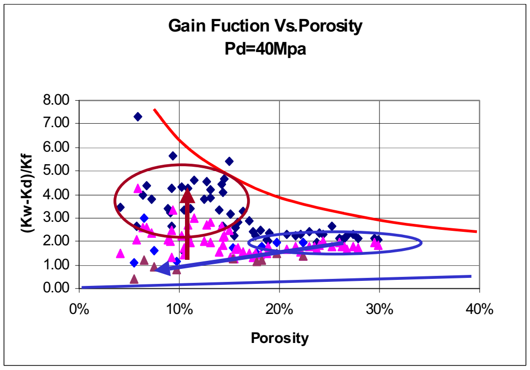

Fig. 5. Gain function for clean and shaly sands: blue dots are based measured ultrasonic data, pink dots are based on Gassmann's calculation.

Data reveal that gain function for reservoir sands (20-30% porosity) is around 2, and slightly decreases with compaction-cementation trend. Data also suggest low dispersion of fluid effects for porous sands. For shaly sands, gain function increases depended on clays (Figure 6), which is consistent with that soft clays are highly stiffened by water.

Gain Function

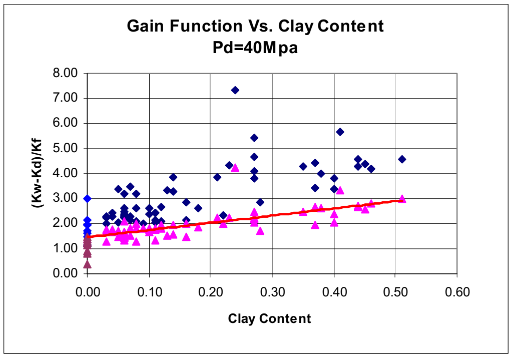

Fig. 6. Gain function as function of clay content: blue dots are based measured ultrasonic data, pink dots are based on Gassmann's calculation.

Fluid Substitution

Fluid substitution is a primary application of Gassmann's equation. With a change of fluid saturation from fluid 1 to fluid 2, the bulk modulus increment is equal to

$$\Delta K_{21}(\leq) \approx G(\phi) \times (K_{f2} - K_{f1}) \qquad (14)$$

where G(ϕ) is the saturation gain function, which remain as a constant as fluid changed. Fluid modulus is a key parameter and provides the fundamental limits to seismic sensitivity to different fluids. Hence, realistic pore fluid properties should be used in any forward modeling (Han and Batzle, 2000a & b).

Lithology Substitution

Lithology discrimination is also a goal for seismic interpretation and the applicability of Equation 14 is usually overlooked. Furthermore, in modeling seismic response, we often must separate fluid influences from lithology effects. To estimate these effects, we can to perform "lithology substitution" by using different gain function as

$$\Delta K_{21}(\leq) \approx (G_2(\phi) - G_2(\phi)) \times K_f \qquad (15)$$

As shown in Equation 13, the gain function is mainly controlled by the ratio of dry rock bulk tomineral modulus. Mineral modulus can have a significant impact on Gassman calculation. Errors due to uncertainty of K_s are less important for clean high porosity (>25%) rocks. Mixed mineral (dirty) and low porosity fractured rocks are more sensitive to K_s . Currently, measurements of the effective modulus of mixed mineralogy rocks are sparse. Alternatively, from measured velocity data on sandstones (Han et al, 1986) we obtain empirical prediction for mineral modulus as show in Table 1.

Table 1. Garin bulk and shear modulus for clean and shaly sands estimated based empirical relation.

Sandstone Modulus (Han, et al., 1986)

Vp = a - b Por.- c*C Vs = a' - b' Por.-c'*C

Pd (bars)	a	b	a'	b'	K(Gpa)	G (Gpa)	b/a	b'/a'
400	5.59	6.93	3.52	4.91	39.0	32.8	1.24	1.39
300	5.55	6.96	3.47	4.84	39.1	31.9	1.25	1.39
200	5.49	6.94	3.39	4.73	39.3	30.5	1.26	1.40
100	5.39	7.08	3.29	4.73	38.7	28.7	1.31	1.44
50	5.26	7.08	3.16	4.77	38.0	26.5	1.35	1.51

Clay	C=0		C=0.1		C=0.2	
Pd (bars)	K(Gpa)	G (Gpa)	K(Gpa)	G (Gpa)	K(Gpa)	G (Gpa)
400	39.03	32.83	37.27	29.40	35.51	26.16
300	39.08	31.91	37.26	28.56	35.44	25.40
200	39.27	30.45	37.30	27.29	35.35	24.30
100	38.74	28.68	36.72	25.73	34.72	22.94
50	38.04	26.46	36.08	23.79	34.15	21.25

Ks = 39.0 Gpa; ΔKs = -1.7 Gpa per 10% Clay

1. For sand with few percent clays, mineral bulk modulus is stable around 39 Gpa if differential pressure higher than 20 Mpa.

2. For shaly sand, clay effect on mineral modulus can be count as -1.7 GPa per 10% clay increment.

Those values suggest that mineral bulk modulus is relatively stable. Clay plays important role in reducing shear modulus but has a more moderate effect on bulk modulus. Data also suggest that few percent of other minerals may be not important to affects mineral modulus for Gassmann' s calculation. However, for rocks with complexly mixed minerals as important part of rock frame the effective grain modulus is still undetermined.

Conclusions

Gassmann's equation is consistent with the both Voigt and Reuss bounds with the minimum and maximum fluid saturation effect on porous rocks. Simplified forms of Gassmann's equations provide an easy understanding of the influences on pore fluid signatures: dry rock gain function

(G) and fluid modulus (K_f) independently control the fluid saturation effect. We need to better characterize and constrain the dry rock properties and use accurate fluid properties before we can correct evaluate fluid saturation effects. Measured data suggest that gain function of reservoir sands is around 2. Clays cause an increase of gain function. The correct grain bulk modulus is another key for accurate fluid substitution, especially for low porosity reservoirs. Empirical relations suggest that grain bulk modulus for clean sandstone is 39 Gpa and decreases with clay content by 1.7 GPa per 10% clay content increment.

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