Pore shape effect on elastic properties of carbonate rocks

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Summary

The presence of non-interactive porosities of spherical or near-spherical type along with microporosity changes the effective elastic properties of the rock frame making pore geometries an important parameter, that must be taken in to account for estimating elastic moduli by any theoretical effective medium models. Differential effective medium (DEM) model is one such theoretical model which accounts for changes in elastic moduli due to changing pore geometries and facilitates inclusion of two or more than two pore shapes.

If bulk porosity and water-saturated P-wave velocities are available, one can estimate the average aspect ratio of the different pore shapes and their relative volume fraction in the rock. These two parameters when used in DEM will predict dry rock moduli and shear velocities. An example on 52 measured carbonate rock samples is shown, in the end.

Introduction

Although velocity-porosity relationship in carbonate reservoirs shows inverse relationship, the measured velocities show lot of scattering in this trend (Figure 1). Carbonate rocks are well cemented; hence grain contact elasticity is not as important as compared to other parameters like mineralogical compositions and pore geometry (Brie et al., 1985). It has been established that the effect of lithological variations is minimal compared to the effect of geometrical properties of pores - shape and size

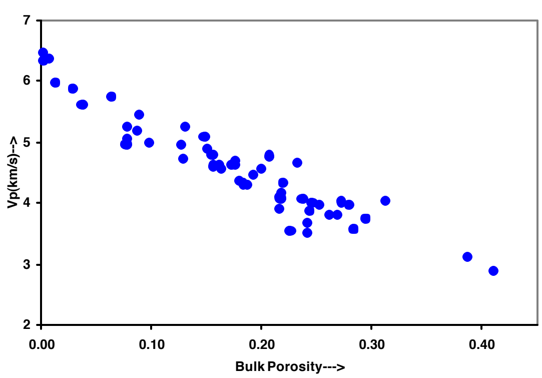

Figure 1: P-wave velocity vs porosity.

(Saleh & Castagna, 2004, Anselmetti and Eberli, 1999). Also, since the wavelength is much larger as compared to the pore size, in both sonic and seismic frequencies, the effect of pore size is negligible when compared to shape of the pores (Brie et al., 1985). Hence, it is fair to assume that scattering on velocity-porosity cross-plot is mainly because of pore-shape effect.

Therefore, an effective medium model which takes into account pore-shape factor is required to estimate the elastic properties of the rock.

Differential Effective Medium (DEM) model

This model simulates porosities in a composite media of two phase by incrementally adding small amount of pores (phase 2) into matrix (phase 1). Berryman (1991) gave a formulation to calculate the effective bulk and shear moduli of this composite media.

$$K(\phi + d\phi) = K(\phi) + \frac{1}{3}[K_f - K(\phi)] \sum_{l=c,nc} \phi_l T_l \frac{d\phi_l}{(1-\phi_l)}$$

$$\mu(\phi + d\phi) = \mu(\phi) + \frac{1}{5}[\mu_f - \mu(\phi)] \sum_{l=c,nc} \phi_l T_2 \frac{d\phi_l}{(1-\phi_l)}$$

With initial conditions K (0) = K_m and μ (0) = μ_m where K_m and μ_m are matrix bulk and shear modulus respectively. K_f and μ_f are the bulk and shear modulus of inclusion phase respectively. ϕ is the porosity and $d\phi$ is the small increment in porosity. T_1 and T_2 are the geometrical factors depending on aspect ratio of the elliptical pores. Aspect ratio of an ellipsoidal pore is defined as ratio of short axis to that of major axis. For fluid saturated inclusions μ_f is 0 and putting K_f in the equation will give dry rock properties of the effective media. The incrementally adding pores to the matrix to generate porosities are a theoretical process and may not be true representation of natural porosity. It's also a path dependent process, for example: In dual porosity type medium adding high aspect ratio spherical pores first and then thin penny-shaped cracks will give different values of elastic moduli than when spherical pores are added afterwards (Figure 2). The difference increases with increasing porosity.

Fluid substitution using Gassmann's equation

Since dry moduli are not asymmetric with respect to inclusion, one way to take out the path dependency of DEM is to use Gassmann's equation for fluid substitution once dry moduli are calculated using DEM. This approach will give us a low frequency estimate of the saturated moduli as compared to DEM's high frequency results. Figure 3 shows a comparison of results obtained using two approaches. The difference in two approaches reduces as the percentage of higher aspect ratio pores increase. For rocks having 100% spherical pores, the two approaches

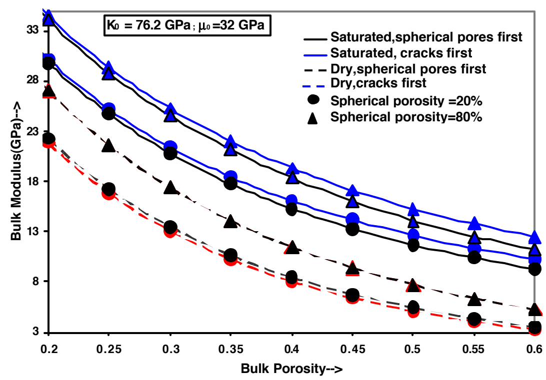

Figure 2: Solid line are saturated bulk modulus and dashed lines are dry modulus. Blue solid lines are DEM results when spherical inclusions are added first, black solid lines are when cracks are added first. Notice that the dry rock modulus do not show path dependence.

will yield the same results. This difference increases with increasing porosity (Figure 3). From this point onwards DEM will refer to this approach of calculating dry moduli and using Gassmann's fluid substitution to obtain saturated moduli.

To use DEM, one requires two inputs which are generally, not available, or hard to obtain from log or measured data. One is aspect ratio of different pore type and another is their relative volume fraction in the rock. It is the objective of this work to obtain average aspect ratio representing each pore type in the rock and their relative volume fraction (porosity).

Methodology

Anselmetti and Eberli (1999) showed that carbonate rocks having only intergranular and intercrystalline porosity (Primary porosity), show little or no deviation in their P-wave velocity from Wyllie time average equation. Inclusion of oomoldic, moldic and vugular porosities cause a positive deviation and effect of microporosity or fractures is to cause a negative deviation from time-average equation. DEM estimates elastic moduli for intergranular porosity close to that predicted by time-average equation for aspect ratio ˜ 0.1. For spherical pores only (aspect ratio ˜ 1) the DEM approximates Hashin-Shtrikman upper bound and for cracks (aspect ratio ˜ 0.01) the DEM approximates Hashin-Shtrikman lower bound (Figure 4).

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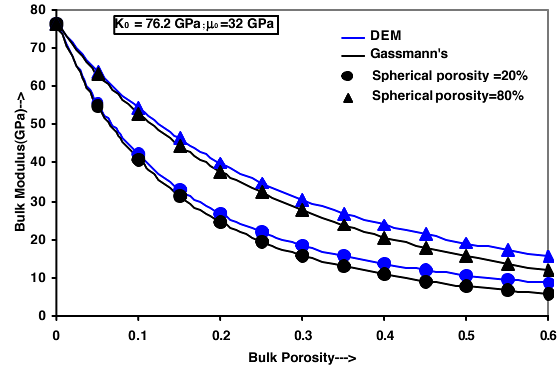

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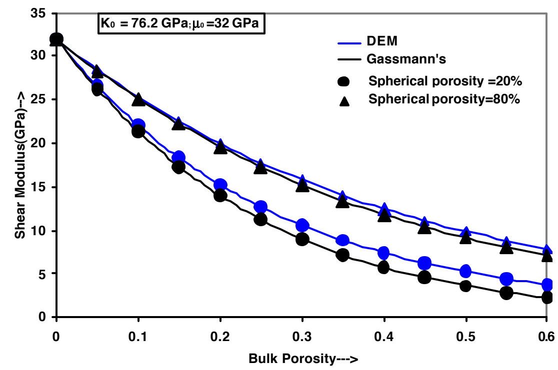

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Figure 3: Blue solid lines are DEM results black solid lines are Gassmann's results. a) Bulk modulus vs bulk porosity b) shear modulus vs bulk porosity.

Estimating aspect ratio (α)

Following steps can be taken to estimate average aspect ratio (α) for each type of pores.

1) For background porosity (intergranular) calculate the P-wave velocity (V_P) using time-average equation for given bulk porosity.
2) For spherical porosity calculate the P-wave velocity (V_P) using Hashin-Shtrikman upper bound for given bulk porosity.
3) For cracks calculate the P-wave velocity (V_P) using Hashin-Shtrikman lower bound for given bulk porosity.

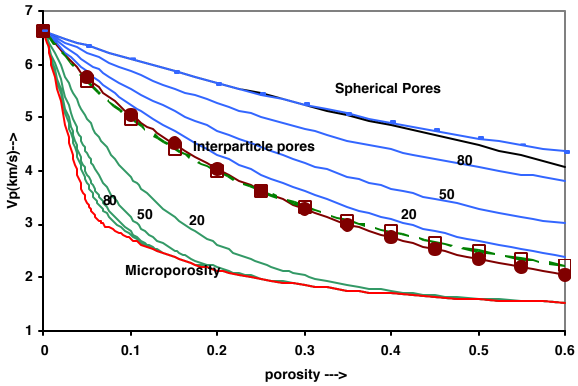

Figure 4: DEM results for different percentage of spherical porosity and cracks along with background porosity of intergranular pores.Solid black line is Hashin-Shtrikman upper boun d and solid red line is Hashin -Shtrikman lower bound. Dashed line is the Wyllie's time-average prediction

4) Use DEM for given bulk porosity and initial estimate of aspect ratio (α = 0.1 for background porosity, α = 1 for spherical pores, α = 0.01 for cracks), to calculate P-wave velocity of the rock. $V_{P,\ DEM} = (K_0, \mu_0, \alpha, \phi_b)$, where K_0 and μ_0 are matrix bulk and shear modulus.
5) If $(V_{P,DEM}(\alpha) - V_P(\alpha))^2 > \varepsilon$ then $\alpha = \alpha \pm \delta\alpha$.
6) Take the aspect ratio (α) for each pore type for which $(V_{P,DEM}(\alpha) - V_P(\alpha))^2 < \varepsilon$.
7) Obtain aspect ratio for primary porosity, spherical porosity and microporosity as α_p, α_s and α_m, respectively.

Porosity estimate

For estimating the effective primary porosity, spherical porosity and microporosity or cracks, one needs an extra input along with the bulk porosity available. This can be any elastic moduli or the P- or S- wave velocities. Usually P-wave velocities are available in form of sonic log, or lab measured data for water saturated core samples. When using sonic logs, care should be taken to input velocities of water-saturated zones only. The main steps in porosity estimation will be:

1) Calculate the $V_{P,Wyllie}$ using time-average equation for given bulk porosity.
2) If measured velocity (V_P) value is greater than $V_{P,Wyllie}$ use $\alpha_1 = \alpha_p$, and $\alpha_2 = \alpha_s$ and $\phi_1 = \phi_b$ and $\phi_2 = 0$.
3) Calculate $V_{P,DEM} = (K_0, \mu_0, \alpha_1, \alpha_2, \phi_1, \phi_2)$.
4) If If $(V_{P,DEM} - V_P)^2 > \varepsilon$ then $\phi_1 = \phi_1 - \delta\phi$ and $\phi_2 = \phi_2 + \delta\phi$.
5) Repeat steps 3 and 4 until $(V_{P,DEM} - V_P)^2 \sim \varepsilon$
6) Obtain effective primary porosity $\phi_p = \phi_1$ and spherical porosity $\phi_s = \phi_2$.
7) If measured velocity (V_P) value is lower than $V_{P,Wyllie}$ use $\alpha_1 = \alpha_p$, and $\alpha_2 = \alpha_m$ and $\phi_1 = \phi_b$ and $\phi_2 = 0$.
8) Repeat steps 3, 4 and 5.
9) Obtain effective primary porosity $\phi_p = \phi_1$ and crack porosity $\phi_m = \phi_2$.

Dry rock moduli and shear velocity

Once all pore type and their relative volume percentage is established DEM can be used to calculate the dry rock elastic properties and shear velocity.

Example

Data were measured on 52 core samples of carbonate rocks from 22 wells, coming from different fields of North America, Middle East and South Asia. These samples come from various depths of 4000 to over 10000 ft. Samples show wide range of porosity ranging from 1 to 40%. Grain density for these samples is around 2.71 gm/cc, a typical value for carbonates. Dry P-and S-wave velocities were measured at various differential pressures on room dried samples. Calculated water saturated velocities using the Gassmann's equations are shown in Figure 5. The measured velocity do not show much variations with varying pressure suggesting the samples contain less soft pores or cracks. Since pressure effect is minimal, we can compile data on samples from different depth, measured at different pressure condition, from different projects to study other parameter effect. The scattering on velocity-porosity plot is then believed entirely due to pore-shape effect. The saturated P-wave velocity is used to put constraint on the inversion of other elastic properties like relative porosities of different pore shapes, dry moduli, and shear velocities. The calculated vs measured data are shown in figure 6. Calculated V_P/V_S ratio for dry and saturated samples is shown in figure 7. As expected, dry V_P/V_S ratio for carbonates do not show much variation with porosity and is constant around 1.9. Saturated V_P/V_S show a decrease with increasing porosity.

Conclusions

With known bulk porosity and water saturated P-wave velocities, one can make an estimate of aspect ratio of different pore type and their relative volume fraction in the rock. This workflow helps in estimating effective primary porosity which is critical to porosity-permeability relationship. One added benefit of using DEM is in the fact that it gives dry rock properties, which may be used to predict shear velocities, not readily available from log measurements. Although, this work is in its initial stages, a good correlation of measured with calculated dry P-wave and S-wave velocities show the promise of this workflow to estimate all the elastic parameters effectively.

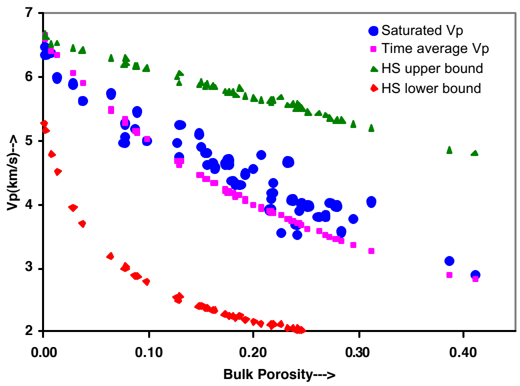

Figure 5: Calculated Vp in the lab from measured dry rock properties and using Gassmann's fluid substitution. Note that most of the samples lie above time-average prediction indicating presence of spherical pores along with background interparticle porosity.

Acknowledgements

We thank Carlos Cobos, Dr. John Castagna, and Dr. Shiyu Xu for their helpful comments.

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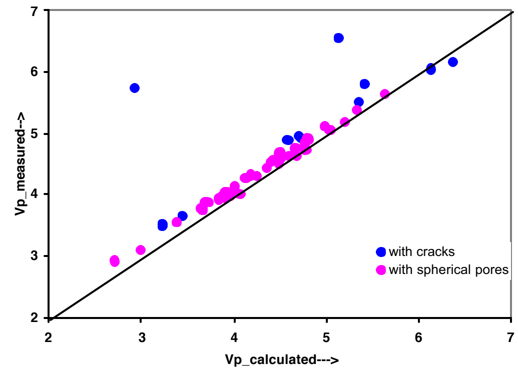

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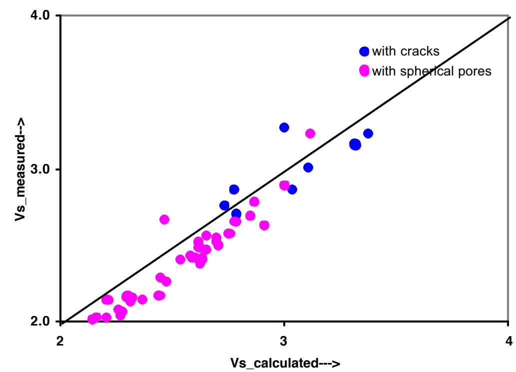

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Figure 6: crossplot of measured and calculated dry P - and S- wave velocities. a) P -wave velocity b)S-wave velocity

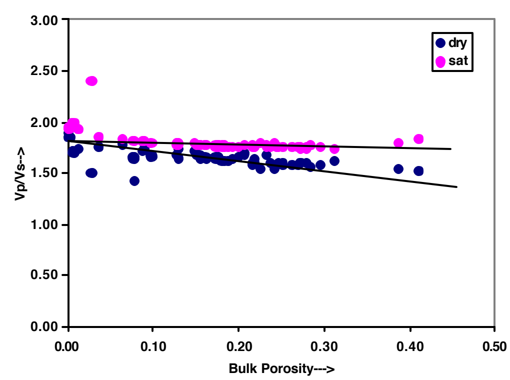

Figure 7: Vp/Vs ratio for dry and saturated (calculated).

EDITED REFERENCES

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